On the abyssal circulation of the world ocean—I. Stationary planetary flow patterns on a sphere

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Abstract—A treatment of stationary planetary flow patterns driven by source-sink distributions in a cylindrical tank (Stommel *et al.*, 1958) is extended to predict flow patterns which might be expected under similar circumstances on a rotating sphere. Flow patterns are sketched for various source-sink distributions and meridional and zonal boundary conditions.

(1) INTRODUCTION

IN a previous paper (STOMMEL *et al.*, 1958) we defined a regime of flow specified by the following elements : (a) the flow in the whole layer is steady and geostrophic except and only (b) at the western boundary where a narrow, intense boundary current is permitted to depart markedly from geostrophy, and (c) the system, which would otherwise be at rest, is driven by a distribution of sources and sinks of fluid (this might include driving agents, such as wind, which can be expressed in terms of source and sink distributions).

Such regimes can occur in a homogeneous layer of fluid (a) of uniform or varying depth on a rotating sphere (b) of uniform or varying depth on a beta plane (c) of radially non-uniform depth on a rotating plane, i.e., in a cylindrical rotating tank with a level bottom. Predictions of patterns of flow deduced from the defined regime have been verified experimentally in the cylindrical rotating tank. It is our purpose here to set forth some theoretical results with respect to circulation patterns which may be deduced for similar flows on a rotating sphere. Where these circulation patterns can be related to existing geometry and boundary conditions in the actual ocean, they might be taken as highly abstracted and idealized models of abyssal ocean circulation. Such models are, of course, highly speculative, since we do not have clear cut confirmation of the existence of the flows described, but we have been encouraged by the success of the rotating tank results to put forth a number of spherical models in order to stimulate further speculation.

In applying this model to the abyssal layer of the ocean, we visualize a distributed sink as involving a transfer of fluid *upward* through the main thermocline, and a concentrated source as a supply of sinking, cold fluid generated in a small area at high latitude.

Our justification for assuming, in these simple models, that there is a general upward flow of water — of several centimetres per day — at mid-depths, is the theory of the oceanic thermocline developed by ROBINSON and STOMMEL (1959). This theory treats each ocean basin as a separate entity. We will show in Part II of this report how a series of such basins can be, in principle, connected together to form a model of the abyssal circulation of the world ocean.

(2) THE FORMULATION OF THE EQUATIONS FOR A HOMOGENEOUS OCEAN

Consider a globe of radius a, rotating with angular velocity ω, covered with a layer of homogeneous water of depth h. Let u be southward component of velocity, and v be eastward component of velocity at a point in co-latitude ϑ and longitude ϕ. (Fig. 1.). Assuming hydrostatic pressure in the vertical, small motions, and absence of an external disturbing force, the linearized dynamical equations are*

$$\frac{\partial u}{\partial t} - 2\omega v \cos \vartheta = -\frac{g}{a}\frac{\partial \zeta}{\partial \vartheta}$$

$$\frac{\partial v}{\partial t} + 2\omega u \cos \vartheta = \frac{-g}{a \sin \vartheta}\frac{\partial \zeta}{\partial \phi}$$

where ζ is the displacement of the free surface.

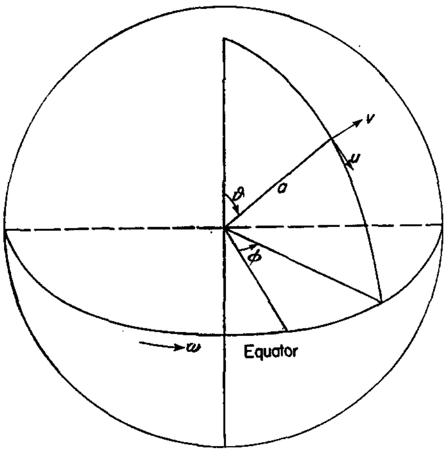

Fig. 1. Notation for co-ordinates and velocities.

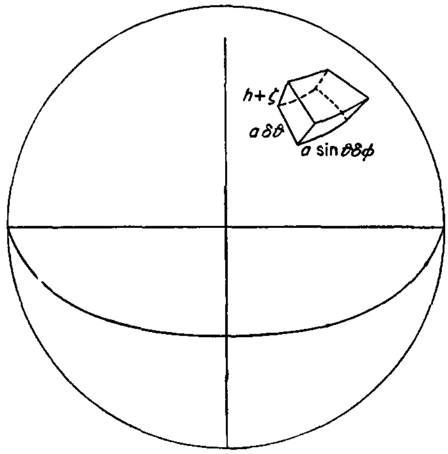

Fig. 2. Element of fluid for formulation of continuity equation.

We must now obtain a suitable form of the continuity equation. Consider a column of water shown in Fig. 2. Neglecting the quantity ζ compared to h, the rate of flow of water out of the column through the vertical walls is

$$\frac{\partial}{\partial \vartheta}(hua \sin \vartheta)\,\delta\phi\,\delta\vartheta + \frac{\partial}{\partial \phi}(hva)\,\delta\phi\,\delta\vartheta$$

At the top of the column, which is of area $a^2 \sin \vartheta\,\delta\phi\,\delta\vartheta$, the free surface is rising at the rate $\partial\zeta/\partial t$, and also there is passage of water through the free surface (positive upward) of $Q(\vartheta, \phi)$ per unit area. (We avoid GOLDSBROUGH's notation $P(\vartheta, \phi)$ because it suggests the word precipitation, of which it is indeed negative, and also looks like Legendre polynomials). We shall call Q the sink function ; it represents a distributed sink.

The total flux upward is therefore $\left(Q + \frac{\partial \zeta}{\partial t}\right) a^2 \sin \vartheta\,\delta\vartheta\,\delta\phi$. Hence the continuity equation is

*We set up the same equations as GOLDSBROUGH (1933).

$$\frac{\partial}{\partial \vartheta}(hu \sin \vartheta) + \frac{\partial}{\partial \phi}(hv) + \left(Q + \frac{\partial \zeta}{\partial t}\right) a \sin \vartheta = 0 \tag{1.2}$$

Let us now consider a steady circulation in a layer of uniform depth. In this case h is constant, and $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial \zeta}{\partial t} = 0$. The solution of (1) and (2) is then

$$u = -\cotan \vartheta \, . \, \frac{Qa}{h} \tag{1.3}$$

$$\frac{\partial v}{\partial \phi} = -\tan \vartheta \, . \, \frac{a}{h} \, . \, \frac{\partial (\cos^2 \vartheta \, . \, Q)}{\partial (\cos \vartheta)} \tag{1.4}$$

$$\frac{\partial \zeta}{\partial \phi} = 2\omega \frac{a^2}{gh} \cos^2 \vartheta \, . \, Q \tag{1.5}$$

An alternate form of (4) :

$$\frac{\partial v}{\partial \phi} = \frac{a}{h} \frac{1}{\cos \vartheta} \frac{\partial}{\partial \vartheta} (Q \cos^2 \vartheta)$$

$$= \frac{a}{h} \left[\cos \vartheta \frac{\partial Q}{\partial \vartheta} - 2 \sin \vartheta \, . \, Q\right] \tag{1.6}$$

(3) EVAPORATION — PRECIPITATION HEMISPHERES WITHOUT MERIDIONAL BOUNDARIES

Consider the globe with no boundaries (coasts). The sink function $Q(\vartheta, \phi)$ is taken as

$$Q = Q_0 \sin \phi \sin \vartheta \tag{2.1}$$

This form of source-sink distribution corresponds to dividing the sphere into a western hemisphere ($\pi \leqslant \phi < 2\pi$) in which Q is negative (precipitation, source) and an eastern hemisphere ($0 \leqslant \phi < \pi$) in which Q is positive (evaporation, sink).

The expressions for u, v, ζ are as follows :

$$u = \frac{Q_0 a}{h} \sin \phi \cos \vartheta \tag{2.2}$$

$$\frac{\partial v}{\partial \phi} = \frac{Q_0 a}{h} [3 \cos^2 \vartheta - 2] \sin \phi \tag{2.3}$$

$$\frac{\partial \zeta}{\partial \phi} = \frac{2\omega a^2 Q_0}{gh} \cos^2 \vartheta \sin \vartheta \sin \phi \tag{2.4}$$

or when integrated

$$u = \frac{Q_0 a}{h} \sin \phi \cos \vartheta \tag{2.5}$$

$$v = \frac{Q_0 a}{h} [2 - 3 \cos^2 \vartheta] \cos \phi + \frac{g}{2\omega a \cos \vartheta} \frac{dG}{d\vartheta} \tag{2.6}$$

$$\zeta = -\frac{2\omega a^2 Q_0}{gh} \cos^2 \vartheta \sin \vartheta \cos \phi + G(\vartheta) \tag{2.7}$$

where $G(\vartheta)$ is an arbitrary function physically associated with an arbitrary zonal flow superposable as an initial condition in the general solution.

The distribution of the sink-function and contours of disturbed height ζ, or simply isobars, are shown in Fig. 3, for $G(\vartheta) = 0$.

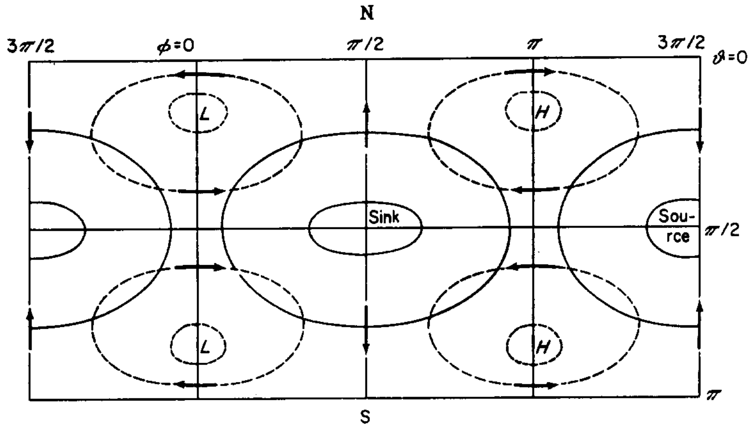

Fig. 3. Circulation pattern for $Q = Q_0 \sin\phi \sin\vartheta$ with no meridional boundaries.

Four self-contained circulation cells exist, separated by the meridian $\phi = 0$, π and by the equator $\vartheta = \pi/2$. The flow is equatorward in the source hemisphere, poleward in the sink hemisphere. The geostrophic planetary flow field is divergent because the Coriolis parameter $2\omega \cos\vartheta$ is a function of latitude. Although all flow is parallel to the isobars, the transport between isobars must vary with latitude. The fields of motion which we derive by the method of GOLDSBROUGH are precisely those whose planetary-geostrophic convergence matches the distributed sink function everywhere.

If we choose the 'step' distribution corresponding to $Q = +Q_0$ on the sink hemisphere ($0 \leqslant \phi < \pi$) and $Q = -Q_0$ on the source hemisphere ($\pi \leqslant \phi < 2\pi$), the solutions of (1.5) and (1.6) are

$$\zeta = \frac{2\omega a^2 Q_0}{gh} \cos^2\vartheta \,.\, \phi + G_1(\vartheta), \qquad (0 \leqslant \phi < \pi)$$

$$= -\frac{2\omega a^2 Q_0}{gh} \cos^2\vartheta \,.\, \phi + G_2(\vartheta), \qquad (\pi \leqslant \phi < 2\pi) \tag{2.8}$$

$$v = -\frac{2Q_0 a}{h} \sin\vartheta \,.\, \phi + \frac{g}{2\omega a \cos\vartheta} \frac{dG_1}{d\vartheta}, \qquad (0 \leqslant \phi < \pi)$$

$$= \frac{2Q_0 a}{h} \sin\vartheta \,.\, \phi + \frac{g}{2\omega a \cos\vartheta} \frac{dG_2}{d\vartheta}, \qquad (\pi \leqslant \phi < 2\pi) \tag{2.9}$$

where G_1 and G_2 must be selected so as to provide continuity of ζ at the boundary between the two hemispheres.

A simple, symmetrical flow pattern analogous to that of Fig. 3 is obtained if we take G_1 and G_2 such as to make $\zeta = 0$ around the great circle meridian $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$.

The result is :

$$\zeta = \frac{2\omega a^2 Q_0}{gh}\cos^2\vartheta \, . \left(\phi - \frac{\pi}{2}\right), \qquad (0 \leqslant \phi < \pi)$$

$$= \frac{2\omega a^2 Q_0}{gh}\cos^2\vartheta \, . \left(\frac{3\pi}{2} - \phi\right), \qquad (\pi \leqslant \phi < 2\pi) \tag{2.10}$$

$$v = \frac{2Q_0 a}{h}\sin\vartheta \, . \left(\frac{\pi}{2} - \phi\right), \qquad (0 \leqslant \phi < \pi)$$

$$= \frac{2Q_0 a}{h}\sin\vartheta \, . \left(\phi - \frac{3\pi}{2}\right), \qquad (\pi \leqslant \phi < 2\pi) \tag{2.11}$$

and u is given by (1.3) with appropriate substitution of $\pm Q_0$ for Q. The circulation pattern is sketched in Fig. 4. There is a singularity at the poles where the velocity u is infinite. Since these conditions are contradictory with the original requirement that the motions be small, we can expect that a higher order dynamical system will be required to explain the detailed configuration at the poles themselves. This points up some of the difficulties which actually could occur in applying the GOLDSBROUGH method to a simple experimental or geophysical system. These difficulties can be avoided, of course, if we are able to use certain simple forms of sink function such

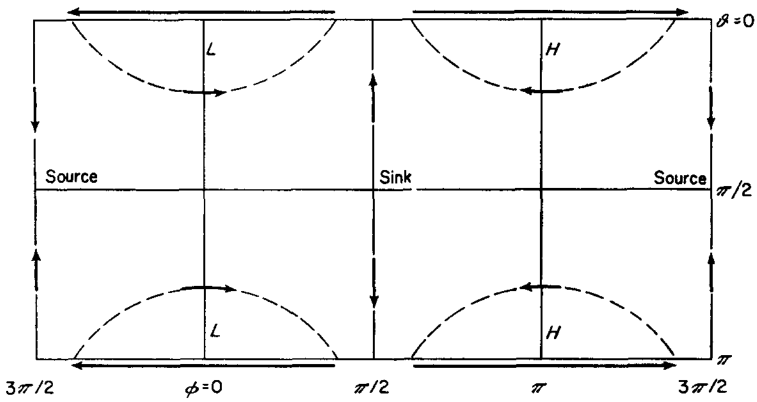

Fig. 4. Circulation pattern for $Q = +Q_0$, $(0 \leqslant \phi < \pi)$, and $Q = -Q_0$, $(\pi \leqslant \phi < 2\pi)$, with no meridional boundaries.

as that employed in Fig. 3. To indicate in Fig. 4 the need for an additional higher order regime at the poles, we resort to a simple symbolic device: We close the isobars in a narrow boundary region at the poles as shown in Fig. 4, so that we do not represent a mathematical discontinuity in pressure, and we draw a heavy arrow at the top to indicate the direction of the singular flow over the poles in this region. The arrows over the lows and highs are both in the same direction, the projection makes them look different. On the sphere, looking at the North Pole, the picture looks something like that shown in Fig. 5.

For simplicity of expression, let us call this little intense stream, the ' polar jet,' or ' polar pinch.'

The polar jet, perhaps surprisingly at first glance, flows from the sink hemisphere to the source hemisphere.

The transport of water in the polar jet must be

$$\frac{gh}{2\omega}(\zeta_{max} - \zeta_{min}) \tag{2.12}$$

From equation (2.8) we find

$$\zeta_{max} - \zeta_{min} = \frac{2\omega a^2}{gh}\pi Q_0 \tag{2.13}$$

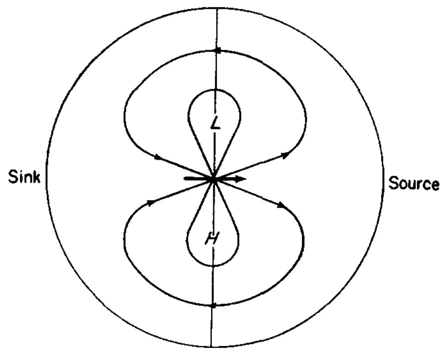

Fig. 5. Circulation pattern of Fig. 4 as seen looking down on the North Pole.

Thus the transport of both the polar jets into the source hemisphere is

$$2\pi a^2 Q_0 \tag{2.14}$$

The total integrated source flowing into the source hemisphere is

$$2\pi a^2 Q_0.$$

Therefore, in the steady state there must be a relatively massive flow, $4\pi a^2 Q_0$, flowing geostrophically from the source hemisphere into the sink hemisphere across the great circle — meridian — bounding the two hemispheres. We may verify this by the following direct calculation: From (2.11) we have for the total zonal flux, F, of material out of the source hemisphere along the great circle meridian $\phi = 0, \pi$:

$$F = 2\pi a Q_0 \sin \vartheta.$$

The total zonal flow out of the source hemisphere is then

$$\int_0^\pi 2\pi a^2 Q_0 \sin \vartheta \, d\vartheta = 4\pi a^2 Q_0,$$

in agreement with the value calculated above from the sum of the transport in the polar jet and the distributed source.

(It might be noted that a somewhat different flow pattern with similar polar jet and zonal transports is obtained if one selects G_1 and G_2 in (2.8) in such a way as to make $\zeta = 0$ along the great circle meridian $\phi = 0, \pi$).

(3) BASINS BOUNDED BY MERIDIANS

We now proceed to discuss the case where an ocean basin is bounded by meridians ϕ_1 and ϕ_2 extending all the way to the poles. Here we develop the pressure pattern from the eastern boundary ϕ_2 where, of course, there can be no zonal transport, so we take $\phi = \phi_2$ as an isobar : $\zeta = 0$. As before, we require geostrophic flow in the interior, but we shall now satisfy continuity requirements by allowing departure from geostrophy in a western boundary current. Our justification for the treatment carried out below rests on the success with which the same method predicts and describes similar flow patterns in a rotating cylindrical tank (STOMMEL *et al.*, 1958).

As an initial illustration we take a uniformly distributed sink Q_0 over the entire surface and balance this by a concentrated source S_0 at the pole, considering the basin north of the equator for simplicity (Fig. 6). Since the surface area of the sector is $a^2(\phi_2 - \phi_1)$, we take the concentrated source at the north pole as $S_0 = Q_0 a^2 (\phi_2 - \phi_1)$. The geostrophic velocities and surface isobars are given by :

$$u = -\frac{Q_0 a}{h} \cot an\, \vartheta$$

$$v = \frac{2Q_0 a}{h} \sin \vartheta \,.\, (\phi_2 - \phi)$$

$$\zeta = \frac{2\omega a^2 Q_0}{gh} \cos^2 \vartheta \,.\, (\phi - \phi_2) \tag{3.1}$$

The pattern of the isobars is shown in Fig. 6. (There is a singularity at the pole similar to that encountered in Section 2).

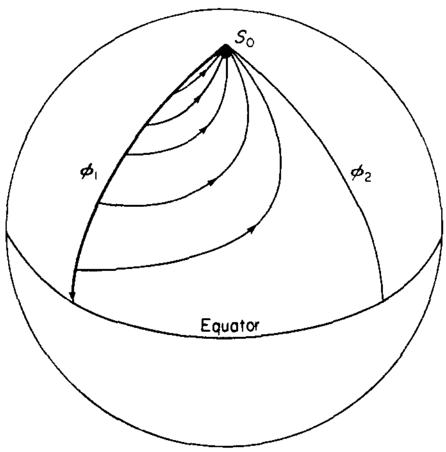

Fig. 6. Circulation pattern in meridionally bounded ocean with concentrated source S_0 at North Pole and a uniformly distributed sink Q_0, such that $S_0 = Q_0 a^2 (\phi_2 - \phi_1)$.

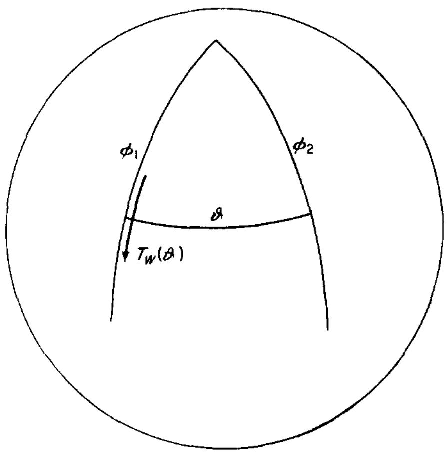

Fig. 7. Notation for evaluating strength of western boundary current $T_W(\vartheta)$ by consideration of continuity of mass flow in a sector.

This flow does not satisfy boundary conditions at $\phi = \phi_1$, and, following the procedure used by (STOMMEL, *et al.*, 1958) we introduce symbolicaly a western boundary current $T_W(\vartheta)$ shown positive southward by the heavy arrow in Fig. 7.

Consider a section of the ocean (Fig. 7) bounded by ϕ_1, ϕ_2, and ϑ. The total outward flux of mass from portions of the basin so bounded consists of four parts :

1. The western boundary current $T_W(\vartheta)$.

2. The flux integrated over the surface area of the basin

$$= \int_0^{\vartheta} \int_{\phi_1}^{\phi_2} Q\, a^2 \sin \vartheta \,.\, d\vartheta\, d\phi.$$

3. The flux across ϑ in the interior of the ocean, away from the western boundary current

$$\int_{\phi_1}^{\phi_2} ahu \sin \vartheta\, d\phi.$$

4. The flux $-S_0$ from any concentrated source present in the section.

In the stationary state these must add up to zero, hence

$$T_W(\vartheta) = -\int_{\phi_1}^{\phi_2} ahu \sin \vartheta\, d\phi - \int_0^{\vartheta} \int_{\phi_1}^{\phi_2} Qa^2 \sin \vartheta\, d\vartheta\, d\phi + S_0 \tag{3.2}$$

Evaluating equation 3.2 in the light of 3.1, we obtain for case of Fig. 6 :

$$\begin{aligned} T_W &= 2Q_0\, a^2\, (\phi_2 - \phi_1) \cos \vartheta \\ &= 2S_0 \cos \vartheta. \end{aligned} \tag{3.3}$$

Thus in the neighbourhood of the pole the transport in the western boundary current, $T_W(0) = 2Q_0\, a^2\, (\phi_2 - \phi_1) = 2S_0$, is *twice* the transport associated with the concentrated source alone, and, as in the cylindrical case developed by STOMMEL *et al.* (1958) a large volume of water is continually being *recirculated* in such a steady-state system.

Equation (3.3) however, reveals a very interesting difference between the spherical and the cylindrical geometries. Where, in the cylindrical basin, the western boundary current does not vary with radial position and, flows undiminished all the way to the outer rim of the tank, the current in the spherical geometry diminishes as it flows to lower latitudes (Fig. 6), dropping to zero at the equator. (Of course, if we assumed a concentrated source S_0 at the pole *larger* than the internal distributed sink flux $Q_0\, a^2\, (\phi_2 - \phi_1)$, the excess would flow across the equator to feed the sector on the other side).

The internal consistency of equations (3.1) and (3.3) is readily checked by evaluating the total zonal transport at $\phi = \phi_1$ and showing it to be equal to the initial total western boundary current $T_W\,(0)$, i.e.

$$\int_0^{\pi/2} hau\, (\phi_1)\, d\vartheta = 2Q_0\, a^2\, (\phi_2 - \phi_1)$$

A somewhat different pattern of western boundary currents arises if we alter the situation in Fig. 6 only by transferring the concentrated source from the pole to the southwest corner of the sector at the equator (i.e. this could stem from a western boundary current crossing the equator from the southern side). We still assume that the source strength S_0 just balances the distributed sink $Q_0\, a^2\, (\phi_2 - \phi_1)$.

If we now evaluate $T_W\,(\vartheta)$ from equation (3.2), remembering that there is no longer a concentrated source within the section being considered in Fig. 7, we obtain :

$$T_W\,(\vartheta) = Q_0\, a^2\, (\phi_2 - \phi_1)\, (1 - 2 \cos \vartheta) \tag{3.4}$$

The interior flow pattern is still defined by (3.1) exactly as it was in Fig. 6, but the nature of the western boundary current is very different and is sketched in Fig. 8.

A current of recirculated water equal to the source strength starts at the pole and flows toward the source just as in the cylindrical case (STOMMEL *et al.*, 1958), but in the spherical geometry this current gradually diminishes to zero at $\vartheta = 60°$ (30° north latitude). A northward current of equal strength starts at the equatorial source

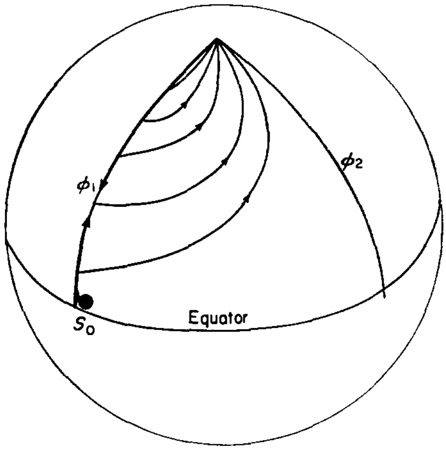

Fig. 8. Circulation pattern in meridionally bounded ocean with concentrated source S_0 (fed by western boundary current from below the equator) and a uniformly distributed sink Q_0 such that $S_0 = Q_0 a^2 (\phi_2 - \phi_1)$.

and also diminishes to zero at 30° north latitude. (This feature is completely absent in the corresponding cylindrical situation). The zonal transport out of these two currents feeds exactly the same interior flow pattern as in Fig. 6.

In the second paper of this series, we shall repreatedly make use of the above results in order to sketch idealized patterns of boundary currents and interior circulation in various basins of the world ocean.

(4) FURTHER ILLUSTRATIONS OF CIRCULATIONS IN MERIDIONALLY BOUNDED BASINS

Applying the analytical method outlined in Section 3, we now state the results obtained for a few other basically interesting cases of source-sink distributions.

(a) A simple form of the function Q which avoids singular flows at the poles is

$$Q = Q_0 \sin^n \vartheta \cos \vartheta, \qquad n \geqslant 1 \tag{4.1}$$

Thus the half of the basin north of the equator is a sink while the southern half is an equal source. S_0 is taken as zero.

The isobars are given by the disturbed height

$$\zeta = \frac{2\omega a^2 Q_0}{gh} \sin^n \vartheta \cos^3 \vartheta \, . \, (\phi - \phi_2) \tag{4.2}$$

and the western boundary current transport by

$$T_W(\vartheta) = Q_0 \, a^2 \frac{\sin^n \vartheta}{n+2} \left[(n+3)\cos^2 \vartheta - 1\right] . \, (\phi_2 - \phi_1). \tag{4.3}$$

The interior isobars and nature of the western boundary current are sketched for the case $n = 0$ in Fig. 9. The north and southbound boundary currents fall to zero at $\vartheta = 54{\cdot}7°$. Here the flow has a singularity at the poles. The singularity is eliminated when $n \geqslant 1$. For larger values of n the flow is essentially similar to that sketched in Fig. 9 with isobars somewhat more concentrated in mid-latitudes.

(b) It is interesting to examine the effect of a sloping bottom, taking $h = h_0 \sin \vartheta$. Here we must go back to the continuity equation in the form of (1.2). For a uniformly distributed sink $Q = Q_0$ and a concentrated source in the south, we obtain :

$$u = -\frac{Q_0\, a}{h_0} \cos \vartheta$$

$$v = \frac{Q_0\, a\, (\phi - \phi_2)}{h_0} (1 - 3 \sin^2 \vartheta)$$

$$\zeta = \frac{Q_0\, a^2\, \omega}{g h_0} (\phi - \phi_2) \cos \vartheta \sin^2 \vartheta$$

$$T_W(\vartheta) = Q_0\, a^2 (\phi_2 - \phi_1) \left[\cos \vartheta\, (1 + \sin^2 \vartheta) - 1\right] \tag{4.4}$$

The isobars and western boundary transport are sketched in Fig. 10. Comparing with Fig. 6, we see that the effect of the sloping bottom is to cause the isobars to intersect the ϕ_1 meridian at different latitudes instead of requiring a common intersection at the pole as in the case of uniform depth. It should also be noted that the southward flowing boundary current first increases in intensity and then decreases instead of showing a monotonic decrease.

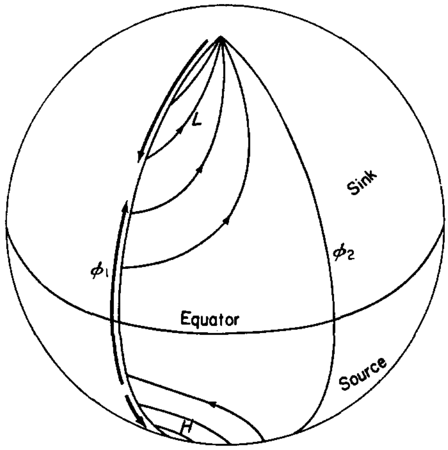

Fig. 9. Circulation pattern for $Q = Q_0 \cos \vartheta$ with no concentrated sources. Note boundary current crossing equator.

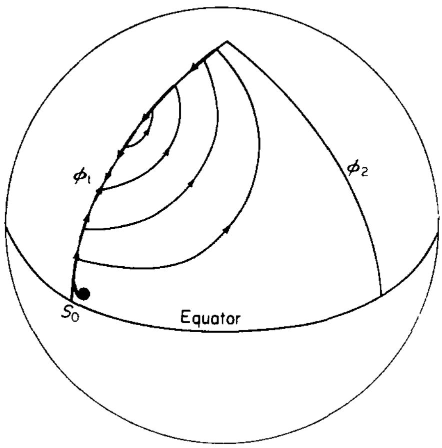

Fig. 10. Circulation pattern in meridionally bounded ocean with sloping bottom. $h = h_0 \sin \vartheta$. Concentrated southern source and uniformly distributed sink.

(c) In any source-sink distribution in which Q becomes zero along a latitude circle, the interior geostrophic flow does not cross this latitude circle, and the circulation in a basin is divided up into separate cells. As an illustration we take

$$Q = -Q_0 \cos 2\vartheta \tag{4.5}$$

giving a distributed source in latitudes higher than 45° and a distributed sink in the belt between the 45° latitude circles. Since the source flux is smaller than the sink flux, we balance the flux by introducing concentrated sources $S_0 = \frac{1}{3} Q_0 a^2 (\phi_2 - \phi_1)$ at each pole. In this case we obtain for a basin of uniform depth :

$$u = \frac{Q_0 a}{h} \cdot \frac{2\cos^3\vartheta - \cos\vartheta}{\sin\vartheta}$$

$$v = \frac{2Q_0 a}{h} (\phi - \phi_2) \sin 3\vartheta$$

$$\zeta = -\frac{2Q_0 a^2 \omega}{gh} (\phi - \phi_2) \cos 2\vartheta \cos^2\vartheta$$

$$T_W(\vartheta) = \frac{Q_0 a^2}{3} (\phi_2 - \phi_1) \cos\vartheta\, (3 - 4\cos^2\vartheta) \tag{4.6}$$

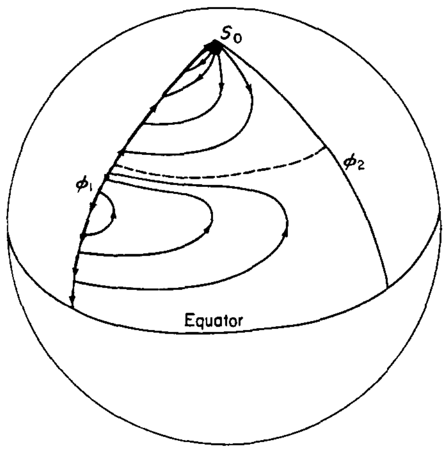

Fig. 11. Circulation pattern divided into separate cells when $Q = 0$ along a latitude circle. $S_0 = \frac{1}{3} Q_0 a^2 (\phi_2 - \phi_1)$; $Q = -Q_0 \cos 2\vartheta$.

The isobars and western boundary transports are sketched in Fig. 11, showing the cells into which the circulation is divided.

(5) BASINS BOUNDED BY MERIDIANS ϕ_1, ϕ_2 AND A NORTHERN LATITUDE ϑ_1 (FIG. 12)

For any distribution of Q, the expressions for u, v and ζ are the same as before, except now it will, in general, be necessary to add a boundary layer at both $\vartheta = \vartheta_1$ and $\phi = \phi_1$. The expression for $T_W(\vartheta)$ must be changed because there is no loss from the sink defined north of ϑ_1 ;
thus

$$T_W(\vartheta) = -\int_{\phi_1}^{\phi_2} ahu \sin\vartheta\, d\phi - \int_{\phi_1}^{\phi_2}\int_{\vartheta_1}^{\vartheta} Q\, a^2 \sin\vartheta\, d\vartheta\, d\phi \tag{5.1}$$

As an example, we may choose $Q = Q_0$, a constant. Hence

$$T_W(\vartheta) = Q_0 a^2 (\phi_2 - \phi_1)(2\cos\vartheta - \cos\vartheta_1) \tag{5.2}$$

The critical latitudes, where the western current reverses direction, are at

$$\vartheta = \cos^{-1}\left(\tfrac{1}{2}\cos\vartheta_1\right) \tag{5.3}$$

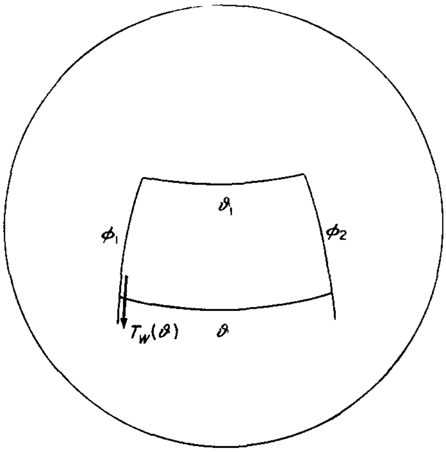

Fig. 12. Notation for sector with rigid boundary at co-latitude ϑ_1.

In the limiting case where the latitude of the northern boundary approaches the pole ($\vartheta_1 \to 0$) the latitude of reversal of the western boundary current is about ($\vartheta = 60°$) 30° N lat.

In some actual applications an additional point source S_0 should be added in the basins, near the poles, to supply the material lost by the distributed sink. In this case we must add S_0 to the equation on the right hand member.

(6) COMPLETELY CLOSED BASIN BOUNDED BY MERIDIANS ϕ_1, ϕ_2 AND LATITUDE CIRCLES ϑ_1, ϑ_2 (FIG. 13)

Again, we take a distributed sink function $Q = Q_0$. The quantities u, $\zeta, T_W(\vartheta)$ are the same as in the case of Section 5, but, of course, for a steady state there must be a source of water somewhere. We place it at the point ϕ_1, ϑ_2. Its intensity must be equal to that of the distributed sink over the whole ocean area.

$$S_0 = \int_{\phi_1}^{\phi_2}\int_{\vartheta_1}^{\vartheta_2} Qa^2 \sin\vartheta \, d\vartheta \, d\phi = Q_0 \, a^2 (\phi_2 - \phi_1)(\cos\vartheta_1 - \cos\vartheta_2) \tag{6.1}$$

The isobar pattern is given by

$$\zeta = \frac{2Q_0 \, a^2 \, \omega}{gh}(\phi_2 - \phi)\cos^2\vartheta \tag{6.2}$$

the pressure pattern being developed from the eastern boundary with zero zonal transport at $\phi = \phi_2$ as before.

In Fig. 13 the isobar pattern according to (6.2) is drawn. It is seen that boundary currents must be drawn at all except the eastern boundary. The western boundary current flows northward from the source, and across the equator, and is met in middle northern latitudes by a southward flowing western boundary current. For example, if we take the latitude of the northern boundary of the ocean 70°N, the latitude of the reversal of the western boundary current is 18·5°N ($\vartheta = 81{\cdot}5°$) from equation (5.3).

If, in the configuration shown in Fig. 13 we now put another point source S_1, at the north-western corner of the basin, and assert that there is a distributed sink whose intensity integrated over the entire area of the ocean is equal to the sum of the two point sources : $S_0 + S_1$, we get a variety of circulation patterns, depending on the relative strengths of sources S_0 and S_1. If they are equal, the flow is in two

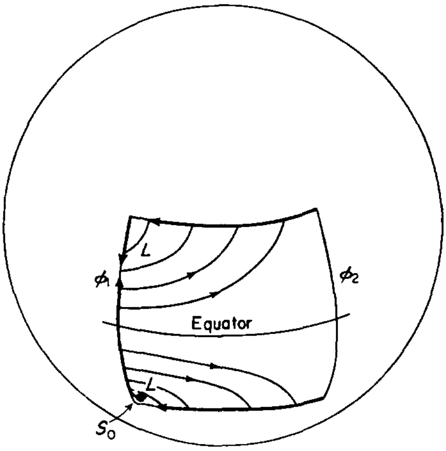

Fig. 13. Circulation pattern in sector bounded by meridians and latitude circles. Boundary currents will be present along north and south boundaries as well as along the western boundary. Single concentrated source.

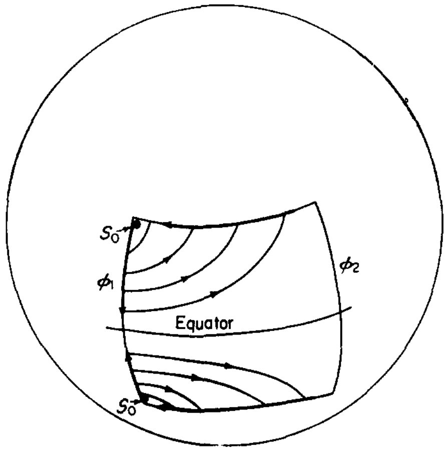

Fig. 14. Circulation pattern in sector of Fig. 13 with equal concentrated sources in each hemisphere.

separate gyres with no flow across the equator (Fig. 14). If the sources are not equal there is a flow across the equator in the western boundary current (there is never transequatorial flow in the interior) from the basin with the larger source, the pattern then looking more like that shown in Fig. 13. The only difference in these patterns, as we vary the relative strength of S_0 and S_1 is in the latitude of reversal of the western boundary current.

(7) SOME COMMENTS ON THE VARIOUS BOUNDARY LAYERS

The various boundary layers introduced in the preceding examples can be avoided, formally, by proper choice of the sink function Q. Thus, all boundary currents on latitude circles can be avoided by choosing Q so that it approaches zero at the boundary. Similarly, western boundary currents can be avoided if we restrict $Q(\vartheta, \phi)$ to forms which satisfy the integral relation

$$\int_{\phi_1}^{\phi_2} Q(\vartheta, \phi)\, d\phi = 0$$

where ϕ_2 and ϕ_1 are the longitudes of the meridional boundaries of the basin. In effect, these are the restrictions which GOLDSBROUGH imposes in his investigation. Such restrictions are, however, quite artificial from a physical point of view. Most simple physical regimes in experiments, such as those in the rotating circular basin with parabolic depth law (STOMMEL *et al.*, 1958) as well as geophysical or oceanographic situations, are certainly not bound to conform to such artificial restrictions. Thus, there is a need to take a rather broad view in choosing models with various forms of Q, and this evidently requires introduction of appropriate boundary layers.

(8) PARTIAL BARRIERS

What will the regime of flow look like when there are meridional barriers which end abruptly at some latitude far from any other coast ? Let us return to the simple configuration originally drawn in Fig. 7, but now we put a long barrier along the longitude $\phi' (\phi_1 < \phi' < \phi_2)$ extending from the pole to colatitude ϑ' but not beyond it (Fig. 5).

We can make use of the same kind of reasoning we used before to construct interior solutions in the open ocean portion. However it is clear that we can now have two western boundary currents along ϕ_1 and ϕ', with transports T_W and T_W'. If upon calculation of the transport T_W' along the partial barrier ϕ' it is found that $T_W' \neq 0$ at $\vartheta = \vartheta'$, then clearly the only resort is a zonal current (Z in Fig. 16) to connect

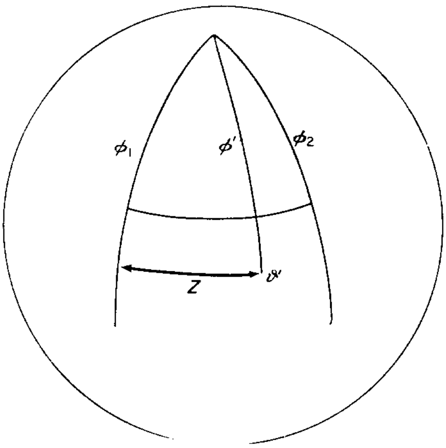

Fig. 15. Meridionally bounded sector (ϕ_1, ϕ_2) with partial meridional barrier extending from pole to longitude ϕ'.

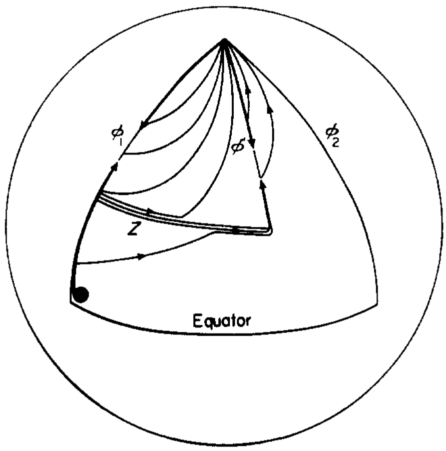

Fig. 16. Circulation pattern in sector with partial meridional barrier. Western boundary currents along ϕ_1 and ϕ' are connected by zonal flow Z.

the western boundary current on the barrier ϕ' to that on the barrier ϕ_1. In this way continuity can be preserved. To see how this alters the picture of the circulation we can compare Figs. 8 and 16. The former shows the pattern of circulation for a basin, bounded by two meridians and the equator, filled by a source S_0 at the equator, and emptied by a uniformly distributed sink over the whole oceanic area. In Fig. 16 a meridional barrier ϕ' has been added part way down from the pole, and the mass flux into the eastern basin occurs partly as a zonal current toward the east. The fact that this zonal current can indeed ' round the corner ' to join the western boundary current has been demonstrated in rotating tank experiments by A. J. FALLER (private communication). Both north and south of the zonal current there is an interior meridional component of flow which must join properly to the zonal current.

Therefore the zonal current is not strictly speaking a continuous current ; water is added from the south and thrown off to the north along its length, as the drawing of the isobars indicates.

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